Particle Data Group entry:

n $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

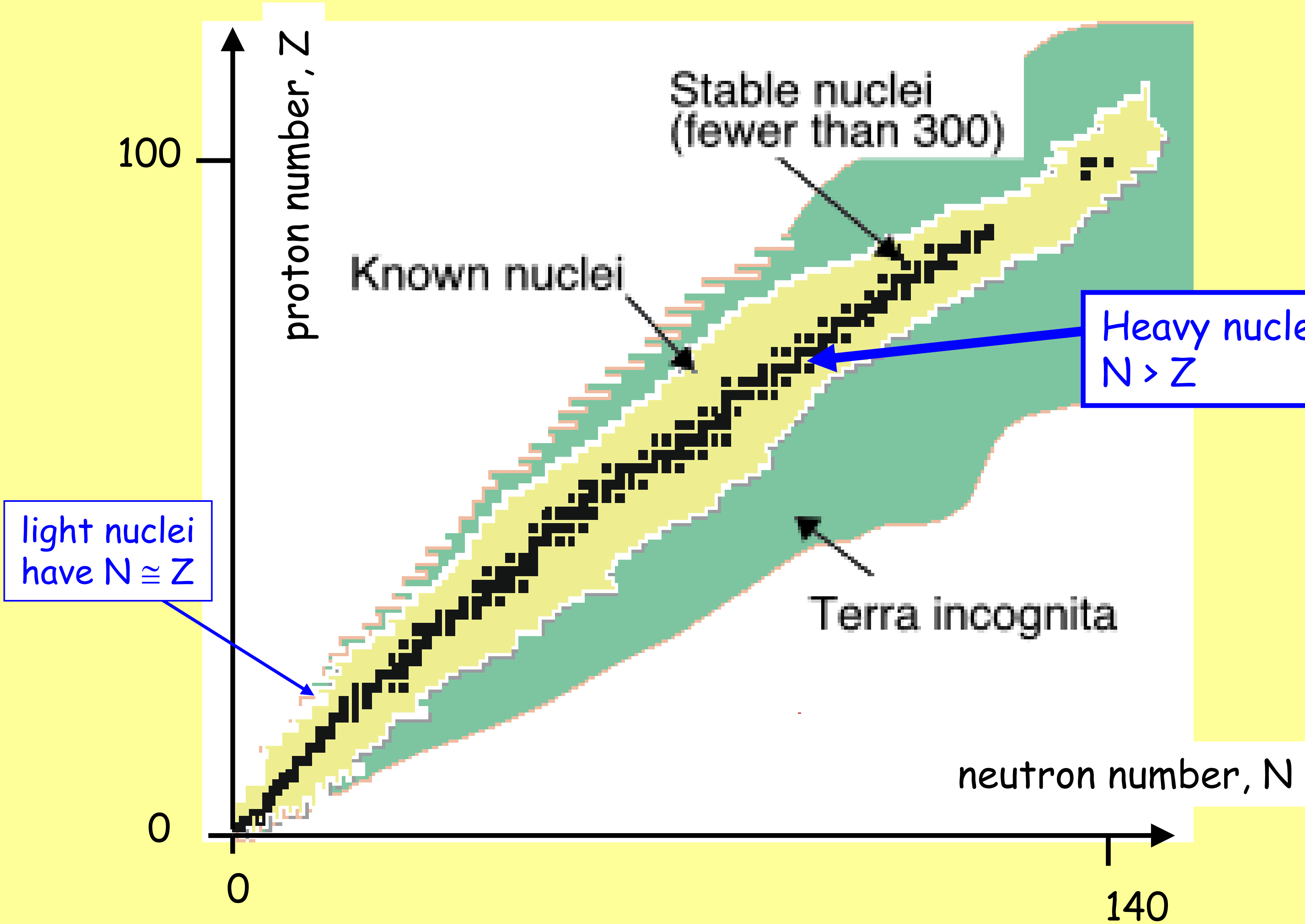
Mass $m = 1.0086649158 \pm 0.0000000006$ u
Mass $m = 939.56533 \pm 0.00004$ MeV [a]
$m_n - m_p = 1.2933318 \pm 0.0000005$ MeV
$= 0.0013884489 \pm 0.0000000006$ u
Mean life $\tau = 885.7 \pm 0.8$ s
$c\tau = 2.655 \times 10^8$ km
Magnetic moment $\mu = -1.9130427 \pm 0.0000005\ \mu_N$
Electric dipole moment $d < 0.63 \times 10^{-25}$ e cm, CL = 90%
Mean-square charge radius $\langle r_n^2 \rangle = -0.1161 \pm 0.0022$ fm^2 (S = 1.3)
Electric polarizability $\alpha = (9.8^{+1.9}_{-2.3}) \times 10^{-4}$ fm^3
Charge $q = (-0.4 \pm 1.1) \times 10^{-21}$ e

???

Note -- contrast to F&H section 6.7 – older data showed $\langle r^2 \rangle \sim 0$

- slightly heavier than the proton by 1.29 MeV *(otherwise very similar)*
- electrically neutral ($q/e < 10^{-21}$!!!)
- spin = ½
- magnetic moment $\mu = -1.91\ \mu_N$ *(should be zero if pointlike: Dirac)*
- unstable, with a lifetime of about 15 minutes: $n \rightarrow p + e^- + \bar{\nu}_e$
- accounts for a little more than half of all nuclear matter

Recall the nuclear "landscape" from lecture 1:



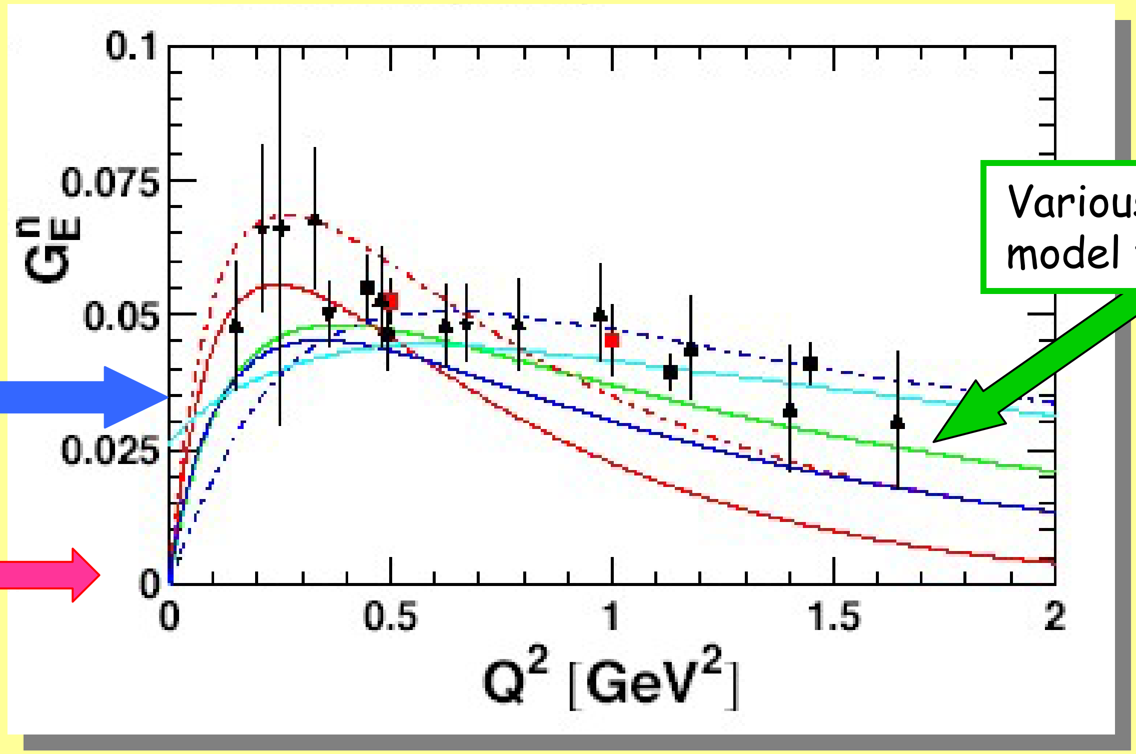
http://www.nscl.msu.edu/future/ria/science/toi.html

Neutron electric form factor: G_e^n *(from elastic electron scattering, etc.)*

- difficult to measure! no free neutron target … (compare ^{1}H and ^{2}H, etc…)
- very small contribution to total cross section, since net charge = 0 (magnetic contribution dominates)
- recall the form factor expansion from lecture 8:

$$F(q^2) = \int \left[1 + i\vec{q}\bullet\vec{r} - (\vec{q}\bullet\vec{r})^2/2 + \ldots \right] \rho(r)\, d^3r = -\frac{q^2\langle r^2\rangle}{6} + \ldots \quad for \quad \int \rho(r)\, d^3r = 0\,!$$

All the world's data (2003):



Positive slope implies negative $\langle r^2\rangle$!

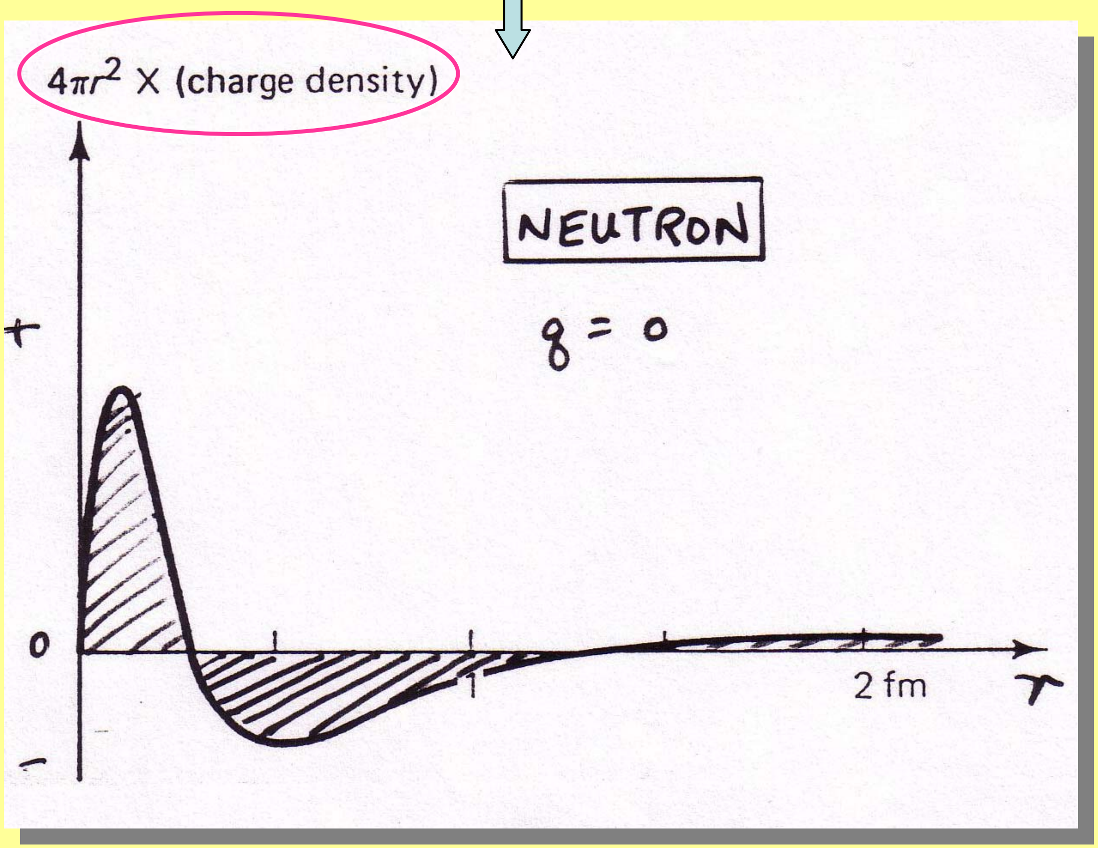
$G_e^n(0) = 0$

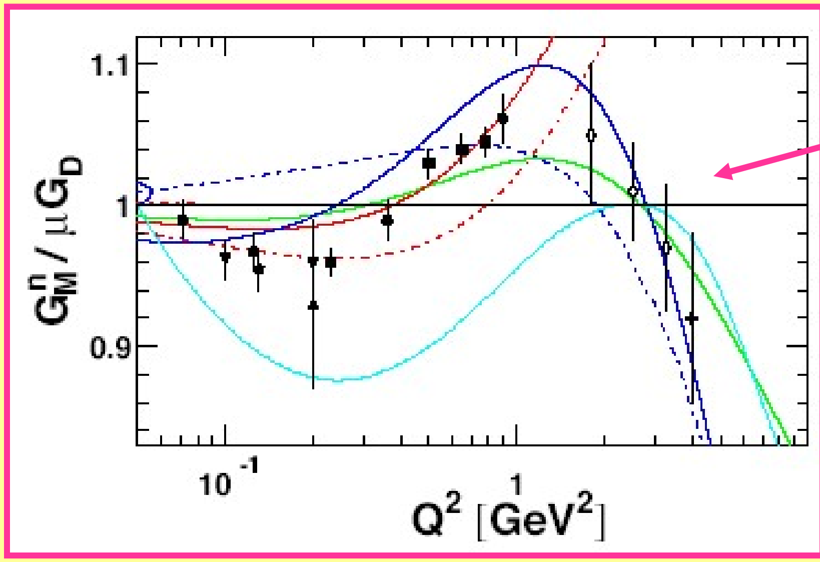
Various quark model theories

What does negative $\langle r^2 \rangle$ mean?

$$\left\langle r^2 \right\rangle \equiv \int r^2 \, \rho(r) \, d^3 r = \int r^2 \, 4\pi \, r^2 \rho(r) \, dr$$

- charge density must have both –ve and +ve regions, since net charge = 0
- integral is weighted with r^2 → more negative charge at large radius

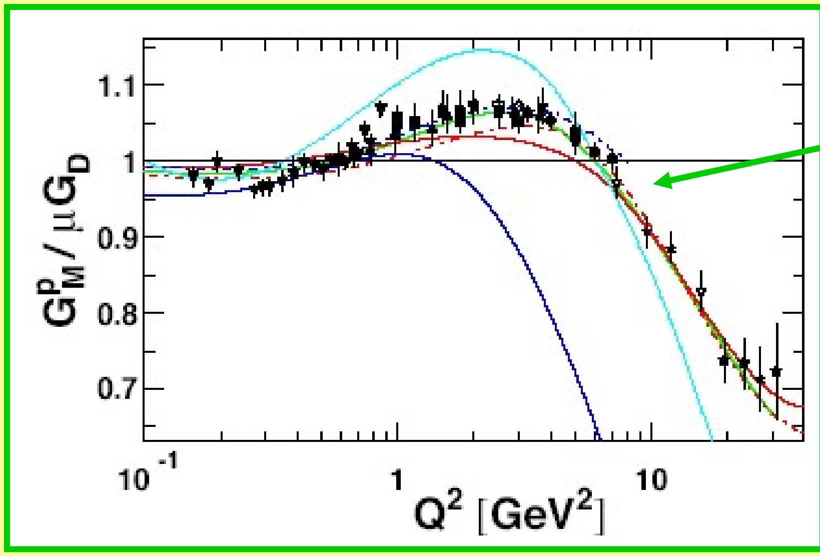




Neutron

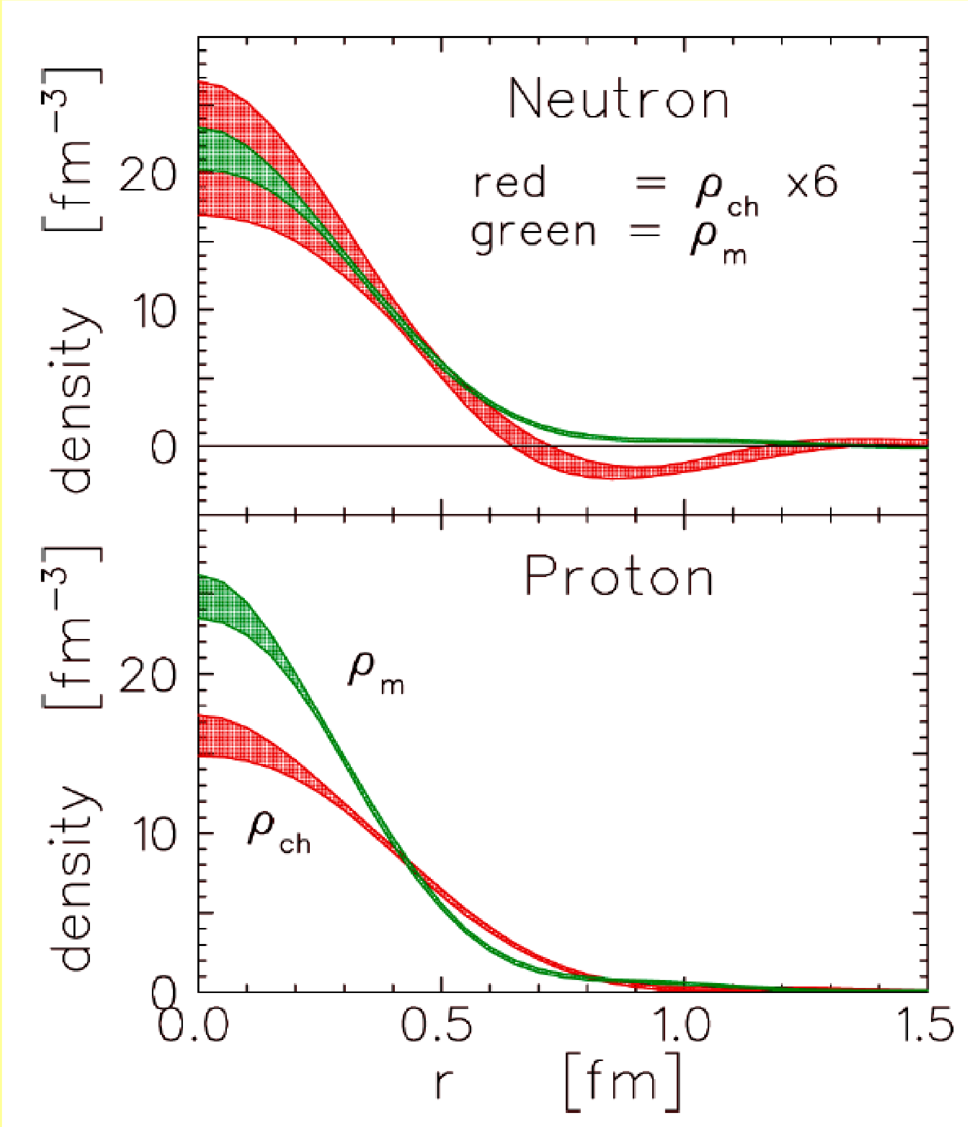
Both plots show ratios to "dipole" fit:

$$G_D = \frac{1}{\left(1 + Q^2 / 0.71\,\mathrm{GeV}^2\right)^2}$$



Proton

Recall: $G_M(0) = \mu$, i.e. the magnetic moment is the "magnetic charge" ...



Kees de Jager: *Nucleon Form Factors* -- talk given at the the 16th International Spin Physics Symposium, spin2004, October 11-16, 2004, Trieste, Italy

Isospin and the nucleon: *(See F. & H, chapter 8; also sec 5.1, 5.2)*

- the neutron and proton are very similar apart from a small mass difference (0.1%) and of course the difference in electric charge
- both play an equally important role in determining the properties of nuclei
- postulate that n,p are two "substates" of a "nucleon", with "Isospin ½", by analogy with ordinary spin s *(Heisenberg, 1932)*

for **spin, S:** $\vec{s} = \frac{1}{2}, \ \langle s^2 \rangle = s(s+1), \ \langle s_z \rangle = m_s = \pm \frac{1}{2}$

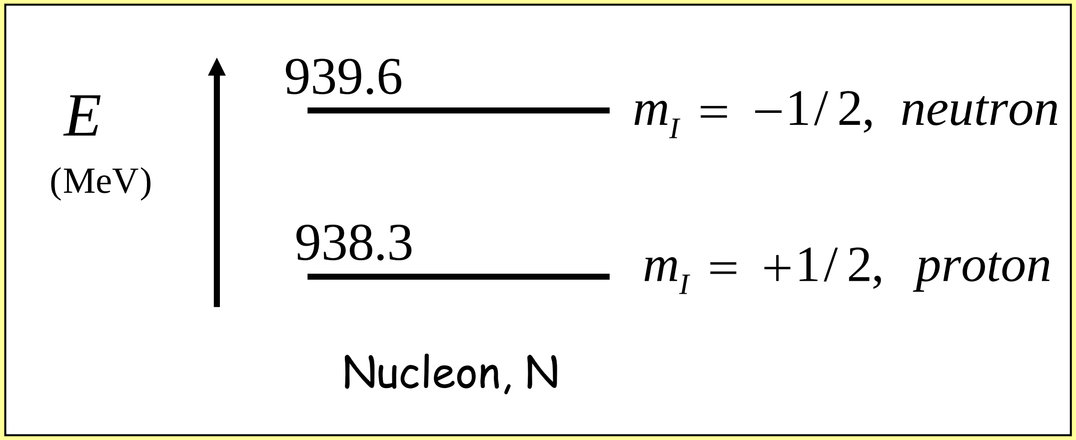
e.g. electron: spin "up" and spin "down" states have different values of m_s, but this is a trivial difference – both are electrons!

→ for **Isospin, I:** $\boxed{\vec{I} = \frac{1}{2}, \ \langle I^2 \rangle = I(I+1), \ \langle I_Z \rangle = m_I = \pm \frac{1}{2}}$

by convention, the **proton has m_I = + ½**, and the **neutron has m_I = - ½**;

these are two "substates" of the **nucleon (N)** with isospin I = ½ !

Nucleon states ($I = \frac{1}{2}$):



- both neutrons and protons have spin $S = \frac{1}{2}$
- S and I are **independent** quantum numbers
- S is "real" in that it has classical analogs in mechanics (intrinsic angular momentum) and electrodynamics (magnetic moment) $\mu = g_s S\, \mu_N$
- I has no classical analog; it is a quantum mechanical vector, literally "like spin" (iso = 'like'), so it follows the same addition rules as S, L, J, etc...
- in this language, (n,p) are isospin-substates of the nucleon, N
- as far as the strong interaction is concerned, $\langle I_z \rangle = m_I$ is all that distinguishes a neutron from a proton

- It turns out to be rather a lucky guess that isospin is a symmetry of the strong interaction: **both m_I and I are conserved** in **strong** scattering and decay processes.
- The electromagnetic interaction breaks isospin symmetry; i.e. it can distinguish between different values of m_I

  $\rightarrow$ There is a simple relation between m_I and electric charge for all **hadrons,** (particles made up of quarks, exhibiting strong interactions...)

Nucleon: N = (n,p) I = ½ isospin doublet, $m_I = \pm \frac{1}{2}$

$\rightarrow$ electric charge $(q/e) = m_I + \frac{1}{2}$ (mass ~ 940 MeV)

Delta: Δ**(1232)** = $(\Delta^{++}, \Delta^{+}, \Delta^{o}, \Delta^{-})$ I = $^3/_2$ isospin quartet, $m_I = (^3/_2, \frac{1}{2}, -\frac{1}{2}, -^3/_2)$

$\rightarrow$ electric charge $(q/e) = m_I + \frac{1}{2}$ (mass ~ 1232 MeV)

Pion or π-meson = (π^+, π^o, π^-) I = 1 isospin triplet, m_I = (1, 0, -1)

$\rightarrow$ electric charge $(q/e) = m_I$ (mass ~ 140 MeV)

Conservation Laws in Quantum Mechanics → see F & H section 7.1, 7.2 !!

<u>Idea</u>: a conserved quantity Q has an expectation value that is constant in time.

$$\frac{d}{dt}\langle Q\rangle = 0$$

since $$\langle Q\rangle = \int \psi^* \, Q \; \psi \;\, d^3 r$$

and $$i\hbar \frac{d\psi}{dt} = H\,\psi$$

$$\Downarrow$$

where H is a time independent Hamiltonian that describes the system, then it follows that:

$$[H, Q] = 0$$

Example: linear momentum in 1-d for a free particle.

$$p_x = -i\,\hbar \frac{d}{dx}; \qquad H = \frac{p_x^2}{2m}; \qquad [H, p_x] = 0 \;\rightarrow\; \langle p_x\rangle = const.$$

$$\psi(x) = \frac{1}{\sqrt{L}}\; e^{ik_x x}, \qquad \langle p_x\rangle = \hbar k_x = const.$$

Application to isospin:

$$\left[H_{strong}, \vec{I}\,\right] = 0 \;\rightarrow\; I \text{ is conserved}$$

$$\left[H_{em}, \vec{I}\,\right] \neq 0 \;\rightarrow\; \text{electromagnetic interaction violates isospin symmetry}$$

A conserved quantity is the same before and after an interaction takes place, e.g.:

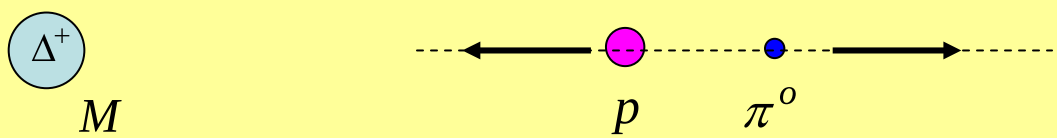
- total energy
- linear momentum
- angular momentum (quantum vector)
- electric charge

from classical mechanics

- parity (exception: weak interaction)
- isospin (strong interaction only)

quantum mechanics

Example: Δ resonance decay, $\Delta^+ \rightarrow p + \pi^o$ in the Δ rest frame:



"before" "after"

Total energy and momentum conservation:

$$M(\Delta) = m(p) + m(\pi) + K(p) + K(\pi), \quad \vec{p}_p + \vec{p}_\pi = 0$$

what about the other quantities? $\rightarrow$

Whether we are adding "spin" or "orbital" or "total" angular momentum (s, l, j), the same rules apply, so we will use "j" in the formalism here:

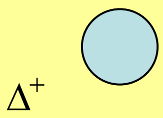
Consider: $\boxed{\vec{j}_1 + \vec{j}_2 = \vec{J}}$

- the total angular momentum has quantum number J and z-projection m_J
- the **z-projections add linearly**: $\boxed{m_{j1} + m_{j2} = m_J}$
- the solutions for J must be consistent with a complete set of configurations m_J, which can be found by writing down all possibilities as above
- this leads to the **general rule**: $\boxed{J = (j_1 + j_2),\ (j_1 + j_2 - 1)\ldots|(j_1 - j_2)|}$
- an exact prescription is beyond the scope of this course, but it involves writing the quantum state $|J,m_J\rangle$ as a linear superposition of configurations $|j_1,m_1,j_2,m_2\rangle$:

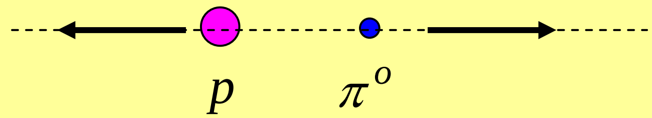
$$\left|J,m_J\right\rangle = \sum_{m_1,m_2} a\,(j_1,m_1,j_2,m_2,J,m_J)\left|j_1,m_1,\ j_2,m_2\right\rangle$$

(The coefficients $a(j_1,m_1\ldots)$ are just numbers; they are called "Clebsch-Gordon" coefficients in advanced books on quantum mechanics.)

Application: $\Delta^{+} \rightarrow p + \pi^{o}$ (the quantum numbers have to add up!)



"before"



"after"

Before:

Angular momentum: J = 3/2

Parity: +

Isospin: I = 3/2, $m_I = \frac{1}{2}$

After:

Angular momentum:

proton: s = ½
pion: s = 0
orbital: L

$$\vec{\frac{1}{2}} + \vec{L} = \vec{J}$$

$$\boxed{L = 1}$$

Parity: proton: +
pion: -
orbital: $(-1)^L$

$$(+)(-)(-1)^L = +$$

Isospin: proton: I = ½, m_I = ½
pion: I = 1, m_I = 0

$$I = (3/2, 1/2)$$

$$m_I = 1/2$$

All the conservation laws are observed. Reaction proceeds in the "I=3/2 channel"

There are a total of 6 quarks in the Standard Model (u,d,s,c,t,b – more later!) but only two play a significant role in nuclear physics: u and d.

Not surprisingly, isospin carries over into the quark description: the "up" quark has isospin I = ½ "up" and similarly for the "down" quark:

Quark "flavor"	Spin, s	Charge, q/e	Isospin projection, m_I
u ("up")	1/2	+ 2/3	1/2
d ("down")	1/2	- 1/3	-1/2

Isospin addition for the proton: p = (uud), $m_I = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2}$ ✓
neutron: n = (udd), $m_I = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = -\frac{1}{2}$ ✓

What about the delta? Addition of 3 x isospin- ½ **vectors:** I = 1/2 or 3/2;
I = 3/2 is the Δ: Δ^{++} = (uuu), Δ^{+} = (uud), Δ^{o} = (udd), Δ^{-} = (ddd) ✓

What about antiquarks? same isospin but **opposite m_I**
→ e.g. pion: (π^+, π^o, π^-)

$$\pi^+ = u\,\bar{d}, \quad m_t = \frac{1}{2} + \frac{1}{2} = +1, \; etc...$$ ✓